

**Protecting Populations**  
**Emphasizing the Importance of**  
**Mathematical Modeling in**  
**Undergraduate Ecology**  
<http://tiny.cc/ecolmodelsmodule>

Kristin McCully<sup>1,2</sup> and Doris Ash<sup>2</sup>  
<sup>1</sup>Ecology and Evolutionary Biology Department,  
 University of California, Santa Cruz  
<sup>2</sup>Education Department, University of California, Santa Cruz  
[kmccully@ucsc.edu](mailto:kmccully@ucsc.edu)

Link will be updated as module is published in other locations

### Outline

- Background
- “Protecting Populations” Module
  - Preparatory Reading
  - Interactive Lecture
  - Cooperative computer inquiry lab
  - Assessment
- Evaluation of module

### Student Quotes

“I didn't really understand why I needed to take math [as a marine biology major] when I would never use it again.”

“I don't remember seeing very many biology examples in my math or biostatistics classes. I saw no link between math and biology.”



“I have not had a lot of math in my biology/ecology classes, but I do know that there is often math, particularly statistics, in the scientific articles we read, so it is definitely good to understand it.”

“I feel like we teach math and then don't use it again, so we need to use math in biology classes and biology in math classes.”

## VISION AND CHANGE

IN UNDERGRADUATE BIOLOGY EDUCATION

A CALL TO ACTION

ADVANCING SCIENCE. SERVING SOCIETY

**Core Competencies and Disciplinary Practice**

**2. ABILITY TO USE QUANTITATIVE REASONING:**  
*Biology relies on applications of quantitative analysis and mathematical reasoning.*  
 All students should understand that biology is often analyzed through quantitative approaches. Developing the ability to apply basic quantitative skills to biological problems should be required of all undergraduates, as they will be called on throughout their lives to interpret and act on quantitative data from a variety of sources.

**3. ABILITY TO USE MODELING AND SIMULATION:**  
*Biology focuses on the study of complex systems.*  
 All students should understand how mathematical and computational tools describe living systems.

p. 14  
[www.visionandchange.org](http://www.visionandchange.org)

**BIO 2010**  
TRANSFORMING UNDERGRADUATE EDUCATION  
 FOR FUTURE RESEARCH BIOLOGISTS

**Recommendations**

1. Given the profound changes in the nature of biology and how biological research is performed and communicated, each institution of higher education should reexamine its current courses and teaching approaches to see if they meet the needs of today's undergraduate biology students. Those selecting the new approaches should consider the importance of building a strong foundation in mathematics and the physical and information sciences to prepare students for research that is increasingly interdisciplinary in character.
2. Concepts, examples, and techniques from mathematics, and the physical and information sciences should be included in biology courses, and biological concepts and examples should be included in other science courses. Faculty in biology, mathematics,

p. 8  
<http://www.nap.edu/catalog/10497.html>

**How?**

- Integrate quantitative approaches through life sciences curriculum (Bialek and Botstein 2004 *Science*, Speth et al. 2010 *CBE Life Sci. Ed.*)
- Restructure math courses for biology majors to incorporate biology (Usher et al. 2010 *CBE Life Sci. Ed.*)
- Completely integrate freshman-level math and biology courses (Matthews et al. 2010 *CBE Life Sci. Ed.*)
- Create quantitative biology majors with capstone course, seminar, or research experience (Usher et al. 2010 *CBE Life Sci. Ed.*)
- Create modules for easy integration into existing COURSES (Nelson et al. 2009 *J. Coll. Sci. Teaching*)

**MathBench Biology Modules**  
 Web-Based Math for All Biology Undergraduates

- Interactive web-based modules introducing mathematical underpinnings of introductory biology
- Informal style with story lines
- Intelligent feedback
- Learner control
- Online quiz on each module for assessment
- 36 modules available online for anyone to use at [www.mathbench.umd.edu](http://www.mathbench.umd.edu)

Nelson et al. 2009 *J. Coll. Sci. Teaching*  
 Thompson et al. 2010 *CBE Life Sci. Ed.*

**Outline**

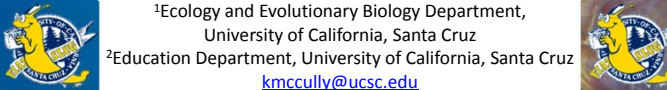
- Background
- “Protecting Populations” Module
  - Preparatory Reading
  - Interactive Lecture
  - Cooperative computer inquiry lab
  - Assessment
- Evaluation of module

# Protecting Populations

## From Ecological Models to Conservation Action

<http://tiny.cc/ecolmodelsmodule>

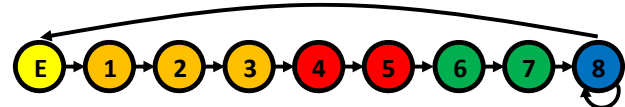
Kristin McCully<sup>1,2</sup>  
<sup>1</sup>Ecology and Evolutionary Biology Department,  
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[kmccully@ucsc.edu](mailto:kmccully@ucsc.edu)



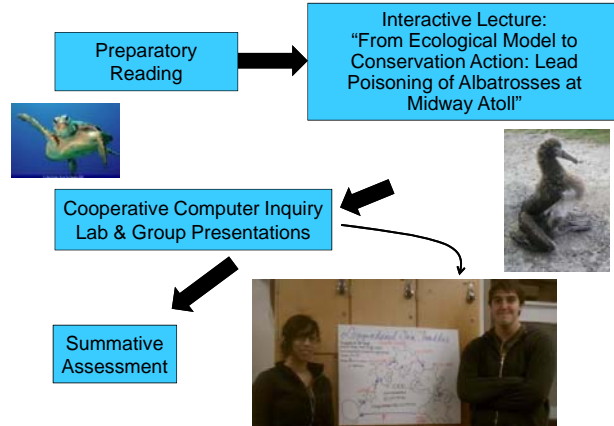
### Learning Goals

- Appreciate the importance and uses of mathematical models in ecology and conservation
- Learn how to use structured population models to assess and protect populations

### Structured Population Models

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}_{t+1} = \begin{bmatrix} P_1 & F_2 & F_3 & F_4 & F_5 & F_6 \\ G_1 & P_2 & 0 & 0 & 0 & 0 \\ 0 & G_2 & P_3 & 0 & 0 & 0 \\ 0 & 0 & G_3 & P_4 & 0 & 0 \\ 0 & 0 & 0 & G_4 & P_5 & 0 \\ 0 & 0 & 0 & 0 & G_5 & P_6 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}_t$$


### Module Design



### Preparatory Reading


Ecology, 68(5), 1987, pp. 1412-1423  
© 1987 by the Ecological Society of America

DEBORAH T. CROUSE  
*Department of Zoology, University of Wisconsin, Madison, Wisconsin, 53706 USA*

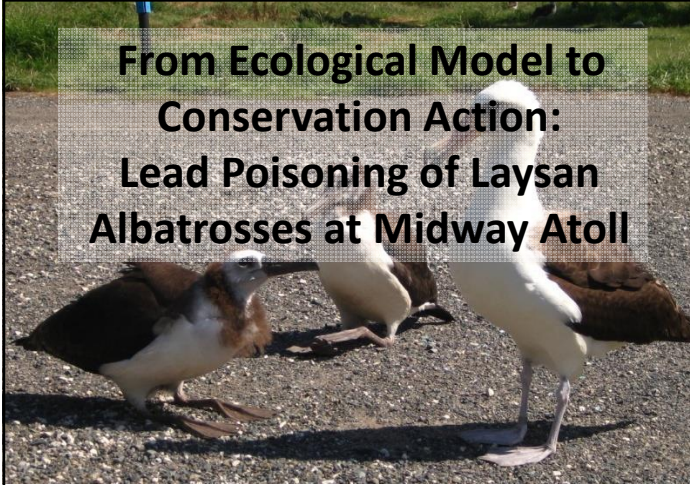
LARRY B. CROWDER<sup>2</sup>  
*Department of Zoology, North Carolina State University, Raleigh, North Carolina 27695-7617 USA*

AND

HAL CASWELL  
*Biology Department, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts 02543 USA*



### From Ecological Model to Conservation Action: Lead Poisoning of Laysan Albatrosses at Midway Atoll



(© 2013 Kristin McCully & Myra Finkelstein)

### Outline

- Introduction to case study:  
Laysan albatrosses at Midway Atoll
- Introduction to mathematical models
- Structured population models
  - Assumptions
  - Model structure
- Mathematical model:  
Laysan albatrosses at Midway Atoll
- Importance of models to ecology and conservation

### Case Study: Lead Poisoning of Laysan Albatrosses at Midway Atoll



Research by  
Myra Finkelstein

**Lead Poisoning of Seabirds:  
Environmental Risks from Leaded  
Paint at a Decommissioned Military  
Base**

**Assessment of demographic risk factors and management  
priorities: impacts on juveniles substantially affect  
population viability of a long-lived seabird**

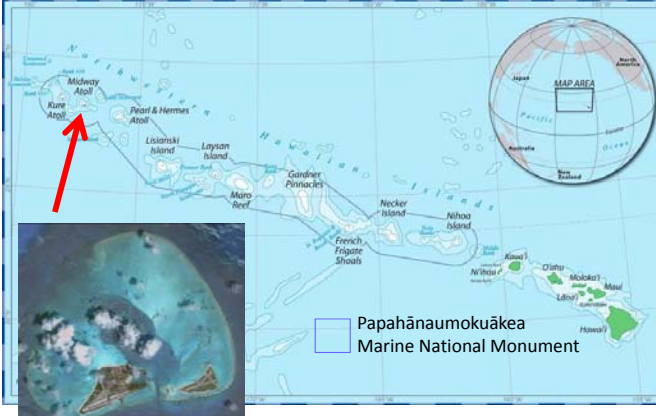
### Threats to Albatross Populations




- Longline fishing bycatch
- Introduced mammals
- Marine debris
- Lead poisoning




### Where is Midway Atoll?

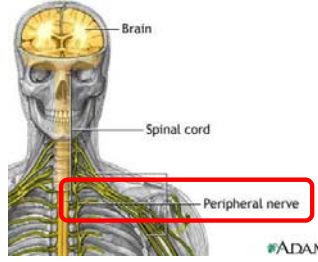


Papahānaumokuākea Marine National Monument

### “Droopwing” syndrome



#### Peripheral Neuropathy




National Institutes of Health

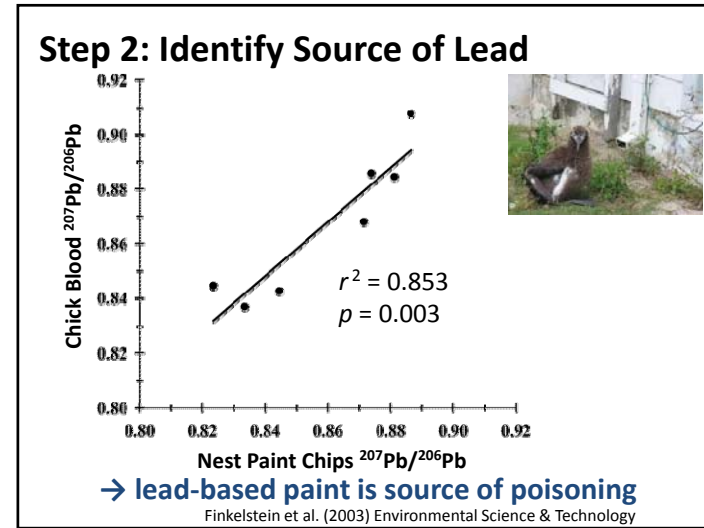
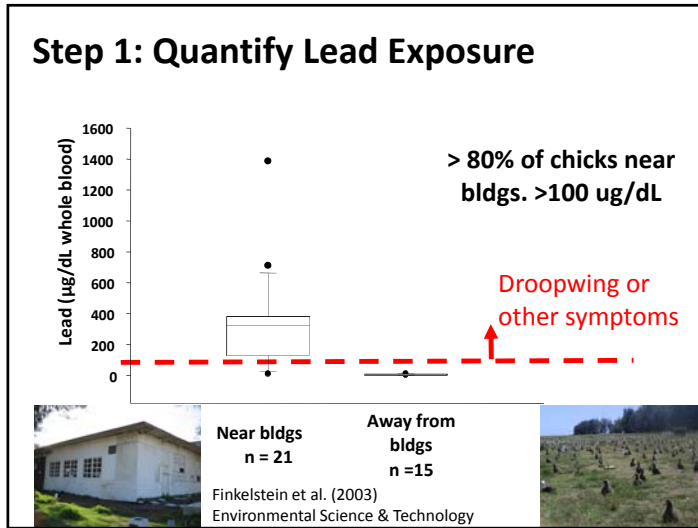
### Problem

#### Lead poisoning of Laysan chicks near buildings

- Sileo and Fefer (1987) J Wildlife Disease
- Sileo et al. (1990) J Wildlife Disease
- Work et al (1996) Env Sci & Toxicology



**Military base → many sources of lead**  
**Need evidence to link source with poisoning**



### Step 3: Show Population Consequences

Mathematical Model

Animal Conservation ZSL

Animal Conservation: Peri ISSN 1367-9420

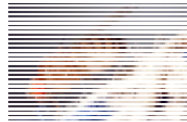
**Assessment of demographic risk factors and management priorities: impacts on juveniles substantially affect population viability of a long-lived seabird**

M. E. Finkelstein<sup>1</sup>, D. F. Doak<sup>2</sup>, M. Nakagawa<sup>3</sup>, P. R. Sievert<sup>4</sup> & J. Klavitter<sup>5</sup>

- ### Outline
- Introduction to case study: Laysan albatrosses at Midway Atoll
  - **Introduction to mathematical models**
  - Structured population models
    - Assumptions
    - Model structure
  - Mathematical model: Laysan albatrosses at Midway Atoll
  - Importance of models to ecology and conservation

### Discussion: What is a Model?

- Simplified representation of a phenomenon
- Requires assumptions



$$\frac{dN}{dt} = \frac{rN(K - N)}{K}$$



Gilbert (2004) Internat. J. Sci. Math. Ed.

### Steps to a Mathematical Model

1. Ask research questions
2. Make assumptions
3. Develop conceptual model
4. Formulate mathematical model
5. Assign values to parameters
6. Use model to answer questions

Based on: Soetaert & Herman 2008 *A Practical Guide to Ecological Modeling* (Fig. 1.7)

### Outline

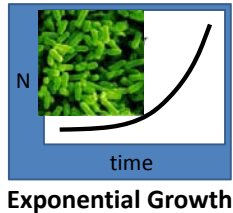
- Introduction to case study:
  - Laysan albatrosses at Midway Atoll
- Introduction to mathematical models
- **Structured population models**
  - Assumptions
  - Model structure
- Mathematical model:
  - Laysan albatrosses at Midway Atoll
- Importance of models to ecology and conservation

### 1. Ask Research Questions

- Is the population growing?
- Which stage(s) and demographic processes should managers focus monitoring and conservation effort on?
- How might threats or management actions impact the population?

## 2. Make Assumptions

- Constant environment (Deterministic)
- Unlimited resources
  - Constant birth & death rates
- No immigration or emigration
- No time lags
- All individuals are identical



From Gotelli (2001) *A Primer of Ecology*

## 2. Make Assumptions

- Constant environment (Deterministic)
- Unlimited resources
  - Constant **vital rates**
- No immigration or emigration
- No time lags
- ~~• All individuals are identical~~
- All individuals in each stage are identical



→ structured population model

From Gotelli (2001) *A Primer of Ecology*

## Structured Population Model

Structured by

- Age
- Size
- Sex
- Developmental stage

Transition Matrix

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}_{t+1} = \begin{bmatrix} P_1 & F_2 & F_3 & F_4 & F_5 & F_6 \\ G_1 & P_2 & 0 & 0 & 0 & 0 \\ 0 & G_2 & P_3 & 0 & 0 & 0 \\ 0 & 0 & G_3 & P_4 & 0 & 0 \\ 0 & 0 & 0 & G_4 & P_5 & 0 \\ 0 & 0 & 0 & 0 & G_5 & P_6 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}_t$$

## Structured Population Model

P = Persistence

G = Growth

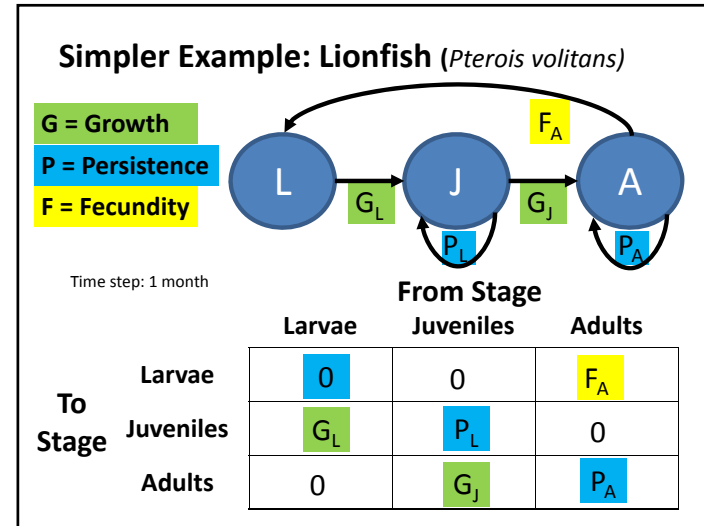
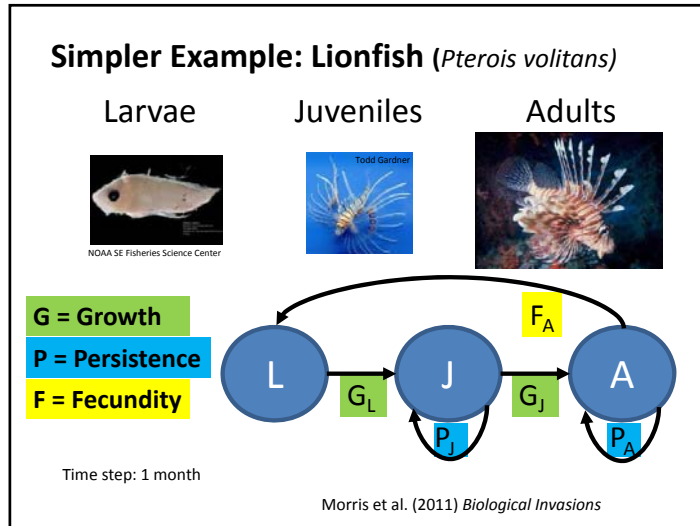
F = Fecundity

FROM


$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}_{t+1} = \begin{bmatrix} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & P_1 & F_2 & F_3 & F_4 & F_5 & F_6 \\ 2 & G_1 & P_2 & 0 & 0 & 0 & 0 \\ 3 & 0 & G_2 & P_3 & 0 & 0 & 0 \\ 4 & 0 & 0 & G_3 & P_4 & 0 & 0 \\ 5 & 0 & 0 & 0 & G_4 & P_5 & 0 \\ 6 & 0 & 0 & 0 & 0 & G_5 & P_6 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{bmatrix}_t$$

TO





- Outline**
- Introduction to case study:
    - Laysan albatrosses at Midway Atoll
  - Introduction to mathematical models
  - Structured population models
    - Assumptions
    - Model structure
  - **Mathematical model:**
    - Laysan albatrosses at Midway Atoll**
  - Importance of models to ecology and conservation


- 1. Determine Research Questions**
- How does lead poisoning of chicks affect population growth?
  - Will lead remediation help protect the population?
- 
- Finkelstein et al. 2009 *Animal Conservation*

### 2. Make Assumptions

- Constant environment (Deterministic)
- Unlimited resources
  - Constant **vital rates**
- ~~• All individuals are identical~~
- No immigration or emigration
- No time lags
- All individuals in each stage are identical

→ **structured population model**

From Gotelli (2001) *A Primer of Ecology*



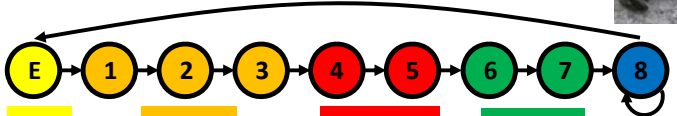
**Complex Life Histories**

### 3. Develop Conceptual Model


**Video: Albatross Life History**




### 3. Develop Conceptual Model




**Eggs**



**Chicks**




**Juvenile**




Pete Leary/USFWS

**Immature**



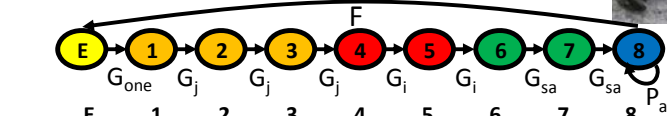
Gina Ruttie

**Adults**



Time step: 1 year

### 4. Formulate Math Model



	E	1	2	3	4	5	6	7	8	F
E										
1	$G_{one}$									
2		$G_j$								
3			$G_j$							
4				$G_j$						
5					$G_i$					
6						$G_i$				
7							$G_{sa}$			
8								$G_{sa}$	$P_a$	

### 4. Formulate Math Model

	E	1	2	3	4	5	6	7	8	F
E	0	0	0	0	0	0	0	0	0	0
1	$G_{one}$	0	0	0	0	0	0	0	0	0
2	0	$G_j$	0	0	0	0	0	0	0	0
3	0	0	$G_j$	0	0	0	0	0	0	0
4	0	0	0	$G_j$	0	0	0	0	0	0
5	0	0	0	0	$G_i$	0	0	0	0	0
6	0	0	0	0	0	$G_i$	0	0	0	0
7	0	0	0	0	0	0	$G_{sa}$	0	0	0
8	0	0	0	0	0	0	0	$G_{sa}$	$P_a$	0

### 5. Assign Values to Parameters

Andrew Derocher (IUCN)

Pete Leary/USFWS

NOAA

NOAA SEFSC

### 5. Assign Values to Parameters

$F = 0.366$

$G_{one}$  0.482

$G_j$  0.84

$G_j$  0.84

$G_j$  0.84

$G_i$  0.906

$G_i$  0.906

$G_{sa}$  0.945

$G_{sa}$  0.945

$P_a$  0.916

**G = Growth**  
**P = Persistence**  
**F = Fecundity**

### 5. Assign Values to Parameters

	E	1	2	3	4	5	6	7	8	F
E	0	0	0	0	0	0	0	0	0	0.366
1	0.482	0	0	0	0	0	0	0	0	0
2	0	0.840	0	0	0	0	0	0	0	0
3	0	0	0.840	0	0	0	0	0	0	0
4	0	0	0	0.840	0	0	0	0	0	0
5	0	0	0	0	0.906	0	0	0	0	0
6	0	0	0	0	0	0.906	0	0	0	0
7	0	0	0	0	0	0	0.945	0	0	0
8	0	0	0	0	0	0	0	0.945	0.916	0

### Steps to a Mathematical Model

1. Ask research questions
2. Make assumptions
3. Develop conceptual model
4. Formulate mathematical model
5. Assign values to parameters
6. Use model to answer questions

Based on: Soetaert & Herman 2008 *A Practical Guide to Ecological Modeling* (Fig. 1.7)

### 6. Use Model to Answer Questions

- A. Is the population growing?  
Population growth projection graph, population growth rate ( $\lambda$ )
- B. Which vital rate(s) should managers focus monitoring and conservation effort on?  
Elasticity
- C. Which stage(s) should managers focus monitoring and conservation effort on?  
Reproductive value
- D. Which stage(s) contain most of the population?  
Stable stage distribution
- E. How might management actions impact population?  
Simulations

### C. Which stage(s) should managers focus monitoring and conservation effort on?

#### Reproductive Value

= expected number of offspring that remain to be born to each individual of a stage

$$v(x) = \frac{\text{\# of offspring produced by individuals of age } x \text{ or older (discounted by likelihood of surviving to reproduce)}}{\text{\# of individuals of age } x}$$

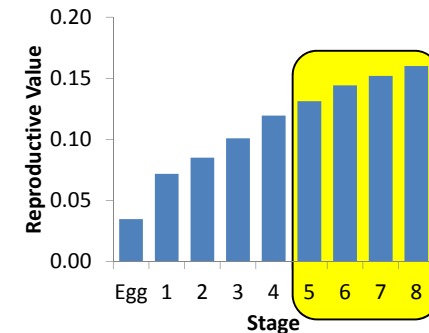
- Essentially which stages are most valuable for future population growth

- Math: left eigenvector of transition matrix
- PopTools: Matrix Tools → Reproductive Value

### C. Which stage(s) should managers focus monitoring and conservation effort on?

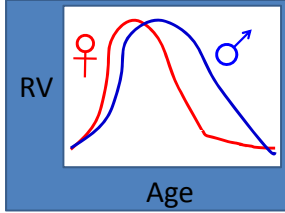


Stage	Reproductive Value
Egg	0.035
1	0.0718
2	0.0851
3	0.1009
4	0.1195
5	0.1314
6	0.1443
7	0.1520
8	0.1601



### Reproductive Value

- Humans?
  - (based on Daly and Wilson 1988 *Homicide*)




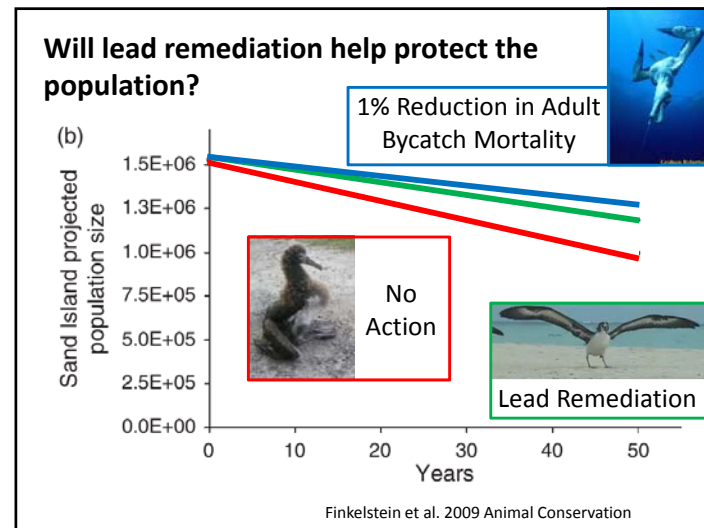
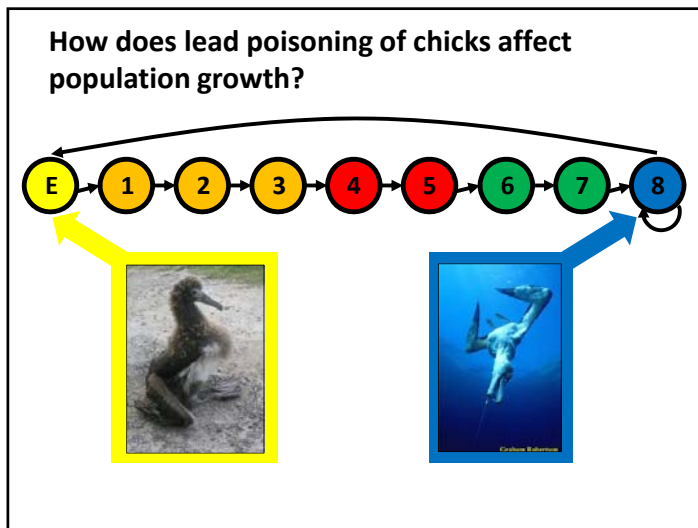
- To maximize sustainable harvest yield, harvest stages with **low** reproductive value
- To maximize impact of restoration, transplant individuals with **high** reproductive value
- Natural selection will act most strongly on stages with **high** reproductive value

Gotelli 2008 *A Primer of Ecology*, Caswell 1980 *Ecology*

### E. How might management actions impact population?

**SIMULATION – What if . . .**

- How does lead poisoning of chicks affect population growth?
- Will lead remediation help protect the population?

## Result: Conservation Action



For Immediate Release, June 18, 2012

**Settlement Will Protect Pacific Seabirds From Lead Poisoning**

**Agreement Requires Cleanup of Lead Paint That Yearly Kills 10,000 Laysan Albatross on Far Pacific's Midway Atoll**



Pete Leary / USFWS

USFWS


For more info: <http://www.fws.gov/midway/lpa.html>

## Discussion:


### How do we use and why do we need mathematical models in ecology and conservation?

## Cooperative Computer Inquiry Lab

The population dynamics of the mangrove *Avicennia marina*; demographic synthesis and predictive modelling




### Ongoing Collapse of Coral-Reef Shark Populations




RECRUITMENT FAILURE, LIFE HISTORIES, AND LONG-TERM DECLINE OF CARIBBEAN CORALS

### Viability Analysis of Reef Fish Populations Based on Limited Demographic Information

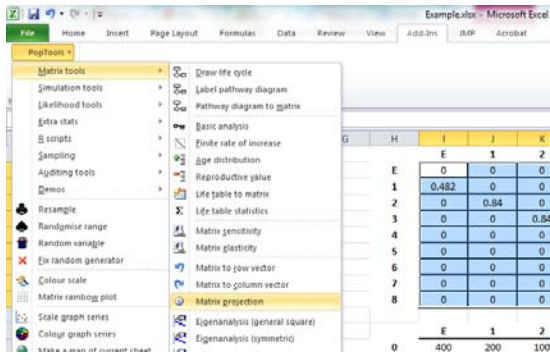


### A STAGE-BASED MODEL OF MANATEE POPULATION DYNAMICS



## Using PopTools Add-on to MS Excel

<http://www.poptools.org/>



	E	1	2
E	0	0	0
1	0.482	0	0
2	0	0.84	0
3	0	0	0.84
4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0
	E	1	2
0	400	200	100

### Lab Worksheet

1. Ask research questions
2. Make assumptions
3. Develop conceptual model
4. Formulate mathematical model
5. Assign values to parameters
6. Use model to answer questions
  - A. Is the population growing (after many years with these vital rates)?
  - B. Which vital rate(s) should managers focus monitoring and conservation effort on?
  - C. Which stage(s) should managers focus monitoring and conservation effort on?
  - D. Which stage(s) contain most of the population (after many years with these vital rates)?

### Lab Worksheet

**Final Challenge:**

Develop and answer your own research question(s) using this data and PopTools. Present your organism’s life cycle and your research question(s) to the class in 3-5 minutes using a written poster.

### Cooperative Computer Inquiry Lab



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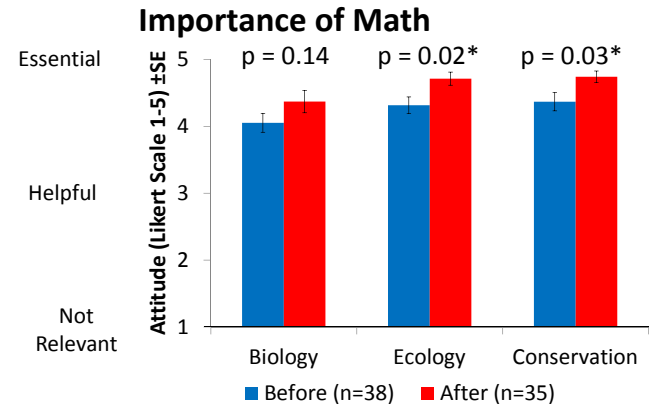
### Student Quotes: Attitude

“The ecological models that we covered in class made me realize how important mathematical models are to understanding population dynamics and conservation biology.”

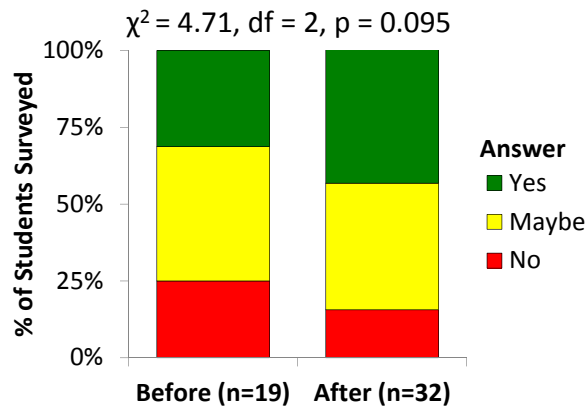
“The model opened my eyes to seeing the importance of math when it came to conservation work. The numbers can display population numbers and help explain why certain animals are endangered compared to others.”

“I assumed you would have to be a math wiz or statistical genius to be able to utilize these types of models and equations in your research (or hire someone to do it for you), but after using PopTools on Excel I realized how easy it is to do yourself.”

### Module Results (2012 & 2013):



### Module Results (2012 & 2013): Take a Quantitative Ecology Class?



### Student Quotes: Learning

“I think the most helpful part of the module was the lab, because we really worked on what the terms, graphs and numbers mean - I think this was more effective in a smaller group.”

“Running the data for ourselves to examine questions we developed was a great exercise [because] not many classes actually have you apply the math concepts you learn.”

“It was fun to interpret the data from the paper we read and to work out the life cycle. I appreciated the math more because it enabled us to answer our own research question, however simple it may have been.”



